Chapter 3: Exponential and Logarithmic Functions

Topic 1: Exponential Functions – p. 438

- Homework: p. 448 & 449 #25-46 even, #53-56

Topic 2: Logarithmic Functions (2 Lessons) – p. 452

- Homework: p. 463 #1-20 even, p. 464 #59-64, 75-80

Topic 3: Log Rules – p. 467

- Homework: Finish #7-12 & 19-24 in notes

Topic 4: Solving Exponential and Log Equations – p. 487

- Homework: p. 488 #1-30 even, p. 489 #49-80 even

Topic 5: Modeling with Exponential/Log Equations – p. 493

- Homework: p. 505 #1-6 even, p. 506 #28-32 even,
  p. 507 #38-42, 47, 48

Name: ____________________ Date: ____________ Period: ______

Chapter 3: Exponential and Logarithmic Functions
Topic 1: Exponential Functions

Do Now:
If $f(x) = 8.39(0.967)^x$

- Evaluate $f(31)$

- Identify the base:
- Graph it in your calculator. What is the y-intercept of the function? (While it's not a whole number, use identifying information in the equation to make a conclusion)

Graphs of exponential functions

Basic Standard form: $\boldsymbol{f(x) = a(b)^x}$

When a and b are positive real numbers:

a: Coefficient. This is the y-intercept of the function in this form. Controls how fast/slow the graph moves. The higher the value of a, the faster the graph increases

b: Base. The base is the multiplier.

- If $0 < b < 1$, the graph is a decreasing graph
- If $b > 1$, the graph is an increasing graph

Full standard form: $\boldsymbol{f(x) = a(b)^{x-h} + k}$

Note: When there are h or k values being added/subtracted on the equation, a is no longer the y-intercept

$x - h$: moves the graph h-units RIGHT

$x + h$: moves the graph h-units LEFT

$+k$: moves the graph k-units UP

$-k$: moves the graph k-units DOWN

Reflections:

$-a$ causes the graph to reflect over the x-axis $-x$ causes the graph to reflect over the y-axis

Examples: Explain the key features of the following equations:

$f(x) = 4\left(\frac{1}{2}\right)^x$ $\quad f(x) = \frac{1}{2}(4)^{x+2} + 7$ $\quad f(x) = -3\left(\frac{1}{3}\right)^{-x} - 4$

The natural base, e

The irrational number, e, is approximately equal to 2.718. This number is called the natural base. When it is the base in an exponential function, it is called the:

Natural Exponential Function: $f(x) = e^x$

Find it on your calculator: e is 2nd→division e^x is 2nd →LN

Example using the natural exponential function.

In one approximation, the population of the world in billions, as a function of years since 1969, is modeled by the function $f(x) = 3.6e^{0.02x}$. In another approximation, the population of the world in billions, as a function of years since 2000, is modeled by the function $f(x) = 6e^{0.013x}$. Calculate the estimated population in 2016 using both models. Which is more accurate to the known current population?

A Review of INTEREST FORMULAS

Compound Interest:

$$A = P\left(1 + \frac{r}{n}\right)^{tn}$$

where n is the number of times the rate is compounded per YEAR

Continuous Compounding:

$$A = P(e)^{rt}$$

Examples:

1. You want to invest \$8000 for 6 years and are choosing between two accounts. The first pays 7% per years, compounded monthly. The other pays 6.85% per year, compounded continuously. Which is the better investment? How much money will you have and the end of 6 year in this account?

2. A sum of \$10,000 is invested at an annual rate of 8%. Find the balance in each after 5 years subject to:

(a) Quarterly compounding

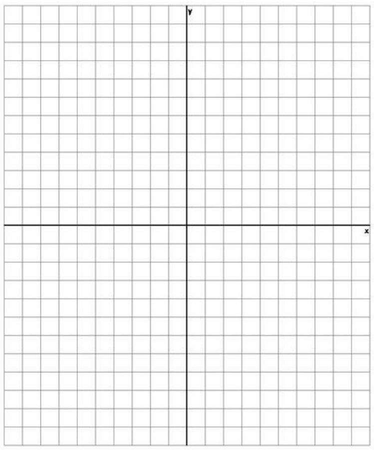
(b) Continuous Compounding

Name: ______________________________ Date: ______________ Period: ______

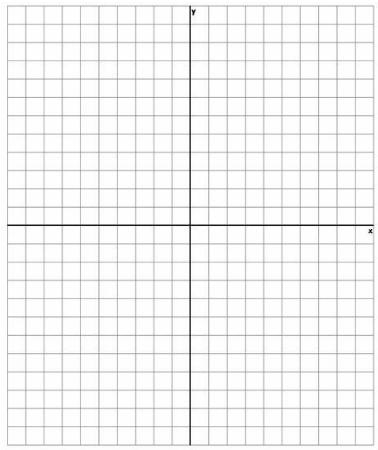
Chapter 3: Exponential and Logarithmic Functions
Topic 1: Homework

Graph the following functions. State all asymptotes, the domain, and the range.

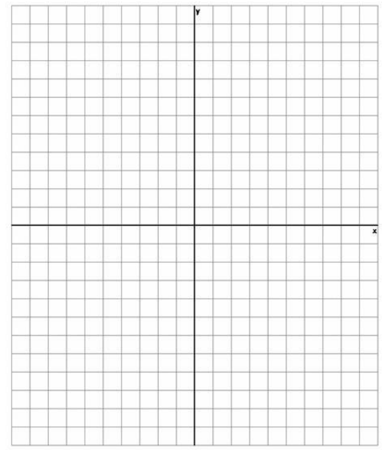
1) $g(x) = 2^x + 1$



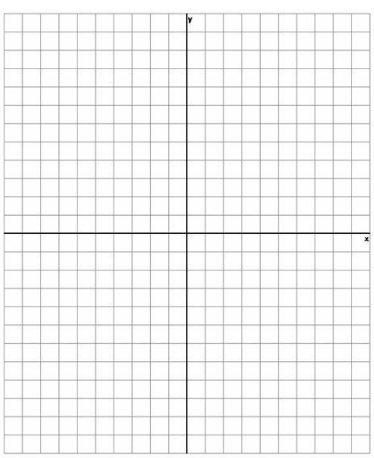
2) $g(x) = 2^{x+1}$



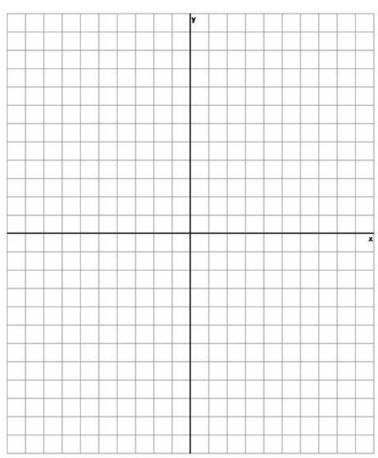
3) $g(x) = -2^x$



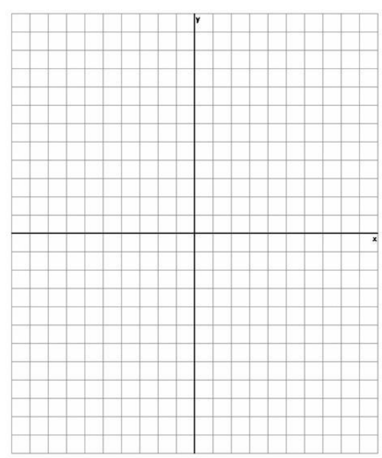
4) $g(x) = 2^{-x}$



5) $g(x) = 2^{x+2} - 1$



6) $g(x) = 3 * 2^x$



Use the compound interest formulas $A = P\left(1 + \frac{r}{n}\right)^{nt}$ *and* $A = Pe^{rt}$ *to solve Exercises 53–56. Round answers to the nearest cent.*

7)

Find the accumulated value of an investment of \$10,000 for 5 years at an interest rate of 5.5% if the money is **a.** compounded semiannually; **b.** compounded quarterly; **c.** compounded monthly; **d.** compounded continuously.

8)

Find the accumulated value of an investment of \$5000 for 10 years at an interest rate of 6.5% if the money is **a.** compounded semiannually; **b.** compounded quarterly; **c.** compounded monthly; **d.** compounded continuously.

Name: ____ Date: ________________ Period: _______

Chapter 3: Exponential and Logarithmic Functions
Topic 2: Logarithmic Functions (Day 1)

Recall: **Logarithm (log) - The power to which a base is raised.**

Logarithmic functions are the INVERSE of Exponential Functions.

Compare and label:

Exponential form

$8 = 2^3$

Log Form

$log_2 8 = 3$

Note: if no 'base' is written, it is implied to be 10.

Practice switching between forms:

	Exponential Form	Log Form
1.	$x = 3^y$	
2.		$3 = log_2 8$
3.	$125 = 5^3$	
4.	$4^2 = 16$	
5.		$3 = log_3 27$
6.	$8^{\frac{1}{3}} =$	
7.		$log_7 1 = 0$

$$b^e = a$$
$$log_b a = e$$

Evaluating in your calculator:
Recall which type of calculator you have

Check: MATH ▸ A: logBASE

If not: input as log a / log b

Evaluate: $log_2 8$ to get an answer of 3

Four Helpful Properties of Logarithms

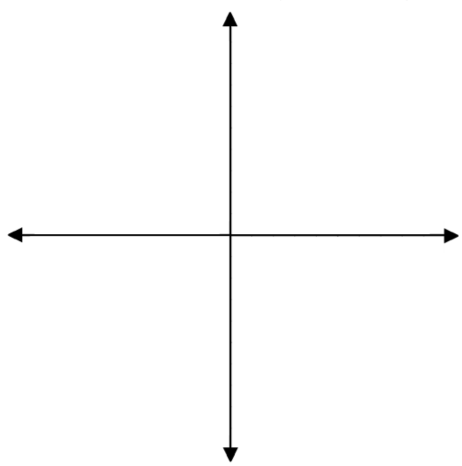
In words	In general form	An example	Switch forms
The log with base b of b, will always result in 1	$log_b b = 1$	$log_7 7 = 1$	
The log with base b of 1, the result must be zero	$log_b 1 = 0$	$log_{17} 1 = 0$	
The log with base b of b raised to a power equals that power	$log_b b^x = x$	$log_4 4^5 = 5$	
b raised to the logarithm with base b of a number equals that number	$b^{log_b x} = x$	$6^{log_6 9} = 9$	

Graphs of Exponential & Log Functions

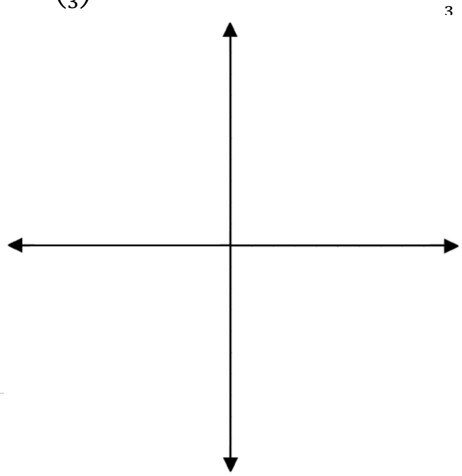
Since log functions are the inverse of exponentials, their paired graphs are reflected over the line $y = x$

Sketch the graph of each exponential or logarithmic function and its inverse:

1. $f(x) = 2^x$ and its inverse $f(x) = log_2 x$ State the domain & range of each



2. $f(x) = \left(\frac{1}{3}\right)^x$ and its inverse $f(x) = log_{\frac{1}{3}} x$ State the domain & range of each



Features of Exponential and Logarithmic Graphs

Feature of a basic function *where $b > 1$*	**Exponential** $\boldsymbol{y = (b)^x}$	**Logarithmic** $\boldsymbol{y = log_b x}$
y −intercept	$(0, 1)$	none
x −intercept	None	$(1, 0)$
Asymptotes	$x - axis$	$y - axis$
Domain	$-\infty < x < \infty$	$0 < x < \infty$
Range	$0 < y < \infty$	$-\infty < y < \infty$

Name: ______________________ Date: ______________ Period: ________

Chapter 3: Exponential and Logarithmic Functions
Topic 2: Homework

For each of the following, convert into the other form. (Log or Exponential).

1. $4 = \log_2 16$
2. $6 = \log_2 64$
3. $2 = \log_3 x$
4. $2 = \log_9 x$
5. $5 = \log_b 32$
6. $3 = \log_b 27$
7. $\log_6 216 = y$
8. $\log_5 125 = y$

9. $2^3 = 8$
10. $5^4 = 625$
11. $2^{-4} = \frac{1}{16}$
12. $5^{-3} = \frac{1}{125}$
13. $\sqrt[3]{8} = 2$
14. $\sqrt[3]{64} = 4$
15. $13^2 = x$
16. $15^2 = x$
17. $b^3 = 1000$
18. $b^3 = 343$
19. $7^y = 200$
20. $8^y = 300$

Name: ______________________________ Date: ______________ Period: ________

Chapter 3: Exponential and Logarithmic Functions
Topic 2: Logarithmic Functions (Day 2)

Modeling with Logarithms

Examples:

1) The percentage of adult height attained by a boy who is x years old can be modeled by:

$$f(x) = 29 + 48.8\log(x + 1)$$

Where x represents the boy's age (from 5 to 15), and $f(x)$ represents the percentage of his adult height. Approximately what percentage of his adult height has a boy attained at age eight?

Approximately what percentage of his adult height has a boy attained at age ten?

2) The magnitude, R, on the Richter scale of a earthquake of intensity I is given by:

$$R = \log\frac{I}{I_0}$$

Where I_0 is the intensity of a barely felt zero-level earthquake. The earthquake that destroyed San Francisco in 1906 was $10^{8.3}$ times as intense as a zero-level earthquake. What was its magnitude on the Richter scale?

Finding Domains of Natural Logarithmic Functions

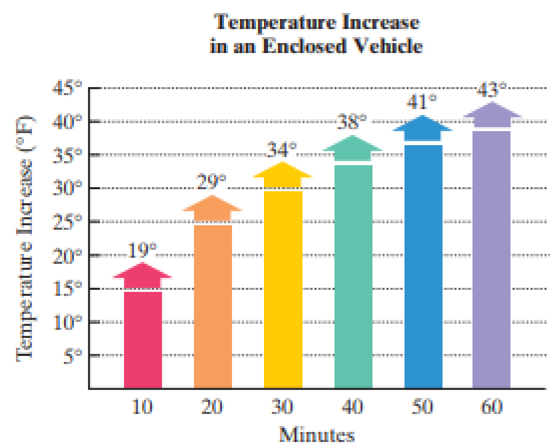
Find the domain of each function:

a. $f(x) = \ln(3 - x)$

b. $h(x) = \ln(x - 3)^2$

c. $f(x) = \ln(4 - x)$

When the outside air temperature is anywhere from 72° to 96° F, the temperature in an enclosed vehicle climbs by 43° in the first hour. The bar graph below shows the temperature increase throughout the hour. The function $f(x) = 13.4 \ln x - 11.6$ models the temperature increase.



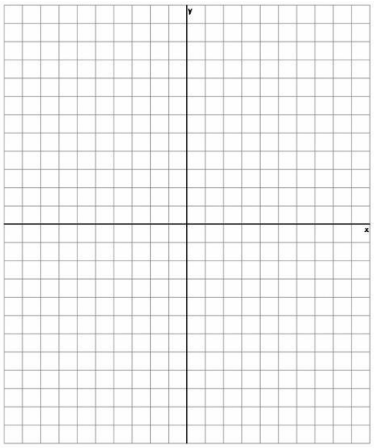
Use the function to find the temperature increase to the nearest degree, after 50 minutes. Compared to the bar graph, how well does the function model the actual increase?

Name: ____ Date: ____________ Period: _____

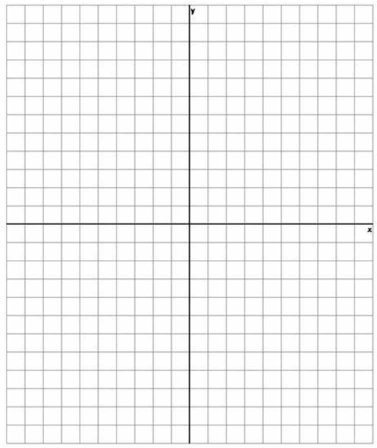
Chapter 3: Exponential and Logarithmic Functions
Topic 2: Homework

Graph the following functions. State all asymptotes, the domain, and the range.

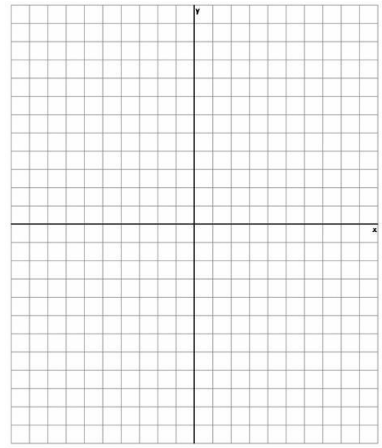
1) $g(x) = \log(x - 1)$



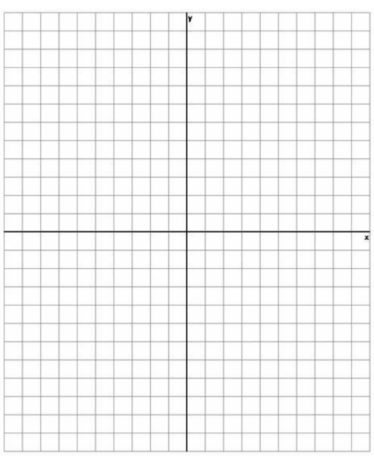
2) $g(x) = \log(x - 2)$



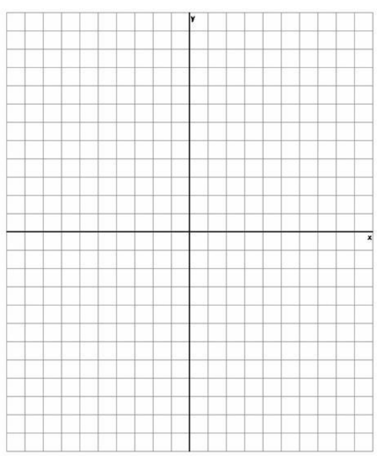
3) $g(x) = \log x - 1$



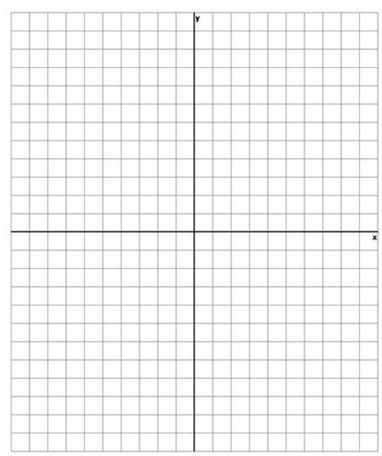
4) $g(x) = \log x - 2$



5) $g(x) = 1 - \log x$



6) $g(x) = 2 - \log x$



7)

The percentage of adult height attained by a girl who is x years old can be modeled by

$$f(x) = 62 + 35\log(x - 4),$$

where x represents the girl's age (from 5 to 15) and f(x) represents the percentage of her adult height. Use the function to solve Exercises 113–114. Round answers to the nearest tenth of a percent.

Approximately what percentage of her adult height has a girl attained at age 13?

Approximately what percentage of her adult height has a girl attained at age ten?

Name: ____ Date: __________________

Note: if no 'base' is written, it is implied to be 10.

Chapter 3: Exponential & Logarithmic Functions
Topic 3: Log Rules

Since logs are exponents, all of the rules of exponents apply to logs as well. Notice the similarities to help you remember!

Rule #1: Product Rule - If two numbers are being multiplied, we add their logs together.

Recall: Exponent Rule: $b^m \times b^n = b^{m+n}$

Log Rule:

Expand:

$log_3(710 \cdot 500) =$

$log_2(8 \cdot 32) =$

Condense:

$log_4 11 + log_4 y =$

$log_5 8 + log_5 2 =$

Rule #2: Quotient Rule - If two numbers are being divided, we subtract the log of the denominator from the log of the numerator

Recall: Exponent Rule: $b^m \div b^n = b^{m-n}$

Log Rule:

Expand:

$log_4 \frac{57}{8} =$

$\log\left(4 \cdot \frac{5}{6}\right) =$

Condense:

$log_4 x - log_4 5 =$

$\log 15 - \log 7 =$

Rule #3: Power Rule - If one number is being raised to a power, we multiple the power by the log of the number.

Recall: Exponent Rule: $(b^m)^n = b^{mn}$

Log Rule:

Expand:

$log_4(10^2) =$

$log(4^2) =$

Condense:

$5log_4 x =$

$\frac{1}{2} log_5(ab) =$

Calculate a numerical answer where possible to check your answers two ways.

Expand: Use the log rules to expand. EVERYTHING gets its own log

1. $log_{12}8x$	2. $log_6 \frac{1}{2}$
3. $log_8 4^x$	4. $log\ 3\sqrt{2}$
5. $log_5(x/z)$	6. $log_3 x^y$
7. $log\ (4xy)$	8. $log\ (a^2 \cdot b/c)$
9. $log_7 10^m$	10. $log \frac{x^2}{b}$

11. $log\ \frac{8^2\sqrt[3]{5}}{21}$ ** hint: rewrite left to right with the radical as a fractional exponent*	12. $\log\frac{x^3y^4}{\sqrt[3]{z}}$ ** hint: rewrite left to right with the radical as a fractional exponent*

Condense: only ONE log in the final answer

13. $2\log m + 3\log n$	14. $\log a - 2\log b$
15. $\frac{1}{2}\log_5 p - 2\log_5 q$	16. $\log m - \frac{1}{3}\log n - \log 5$
17. $2(\log_3 x + 5\log_3 y) - \log_3 z$	18. $2\log g + 3\log h - \frac{1}{2}\log k$
19. $3\log k - (5\log p + \log c)$	20. $5\log_a g + 4\log_a y + 3\log_a h$
21. $(5\log a + \log b) - \log c$	22. $(\log_2 x + \log_2 y) - (\log_2 z + \log_2 m)$
23. $100\log 4 + 2\log 5$	24. $2(\log_6 7 - \log_6 4)$

Name: ______________________________ Date: ______________ Period: ______

Chapter 3: Exponential & Logarithmic Functions

Topic 4: Solving Exponential & Log Equations

<u>We are NOT following the textbook's method on this topic AT ALL!</u>

Solving Exponential Equations

Variable stuck in an exponent? *Switch* to logarithmic form to solve

Examples: To the nearest hundredth if necessary

1. $4^x = 15$

2. $40(e)^{0.6x} = 240$

3. $5^{4x-7} - 3 = 10$

When solving log equations, you MUST check your answers for extraneous roots. Solutions can be negative… but we can't take the log of a negative!

Solving Logarithm Equations

Variable stuck in a logarithm? *Switch* to exponential form to solve

Examples: To the nearest hundredth if necessary

4. $log_4(x+3)=2$

5. $3\ln(2x)=4$

6. $log_2 x + log_2(x-7)=3$

But what if there's logs on both sides?

Condense fully, and drop the logarithm

7. $log_4 4x + log_4 x = log_4 64$

8. $3log_2 4 = log_2 x$

9. $log_{16}(p^2 - p + 4) - log_{16}(2p + 11) = \frac{3}{4}$ *Remember this one?*

Name: ______________________________ Date: ______________ Period: ______

Chapter 3: Exponential & Logarithmic Functions
Topic 4: Homework

Solve each of the following equations.

1. $2^x = 64$

2. $3^x = 81$

3. $5^x = 125$

4. $5^x = 625$

5. $2^{2x-1} = 32$

31. $3e^{5x} = 1977$

32. $4e^{7x} = 10{,}273$

33. $e^{1-5x} = 793$

34. $e^{1-8x} = 7957$

35. $e^{5x-3} - 2 = 10{,}476$

49. $\log_3 x = 4$

51. $\ln x = 2$

53. $\log_4(x + 5) = 3$

55. $\log_2(x + 25) = 4$

57. $\log_3(x + 4) = -3$

77. $\log(x + 4) = \log x + \log 4$

78. $\log(5x + 1) = \log(2x + 3) + \log 2$

79. $\log(3x - 3) = \log(x + 1) + \log 4$

80. $\log(2x - 1) = \log(x + 3) + \log 3$

Name: ______________________________ Date: ______________ Period: ____

Chapter 3: Exponential & Logarithmic Functions
Topic 5: Modeling with Exponential & Log Functions

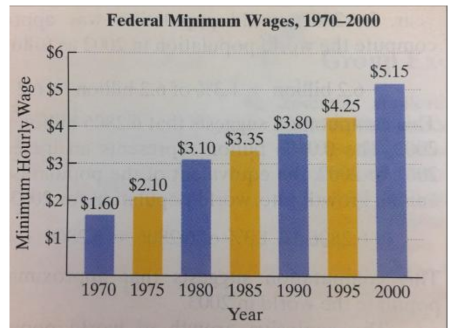
Exponential Growth & Decay Model $A = P(e)^{rt}$

In these questions, other pieces may be missing instead of just plugging in!

Example: The graph shows the growth of the minimum wage from 1970 through 2000.

a. Find the exponential growth function that models the data for 1970 through 2000.

Consider x-values for the years - what is the principal (starting) value?
Use any known point to solve for r



b. By which year will the minimum wage reach $7.50 per hour?

Example: In 2000, the population of Africa was 807 million and by 2011, it had grown to 1052 million.

a. Use the exponential growth model $A = A_0 e^{kt}$, in which t is the number of years after 2000, to find the exponential growth function that models the data.

b. By which year will Africa's population reach 2000 million, or two billion?

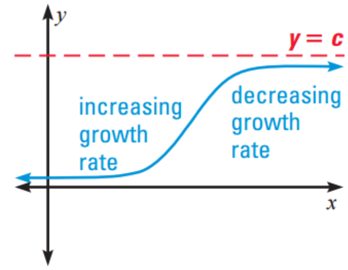
Exponential Growth & Decay Model $\boldsymbol{A = P(e)^{rt}}$

Example: The half life of Carbon-14 is 5715 years. That is, after 5715 years, a sample of Carbon-14 will have decayed to half of the amount. What is the exponential decay model for Carbon-14?

- Begin with the exponential decay model.
- In terms of P, what will the output (A) be?
- Divide both sides by P
- Log to solve for k

Logistic Growth Model $\quad \boldsymbol{A = \frac{c}{1+a(e)^{-bt}}}$

Where a, b and c are constants with $c > 0$ and $b > 0$



Unlike exponential growth which has no upper bound (can increase infinitely), logistic growth does have an upeer bound. As time increases, the expression $a(e)^{-bt}$ approaches zero, making A approach c. Therefore, ***c* is the limit to the growth of A** and creates a horizontal asymptote for the graph of these functions

Example: The function $f(t) = \frac{30000}{1+20e^{-1.5t}}$ describes the number of people who have become ill with influenza t weeks after its initial outbreak in a town with 30,000 inhabitants.

How many people became ill with the flu when the epidemic began?

How many people were ill by the end of the 4th week?

What is the limited size of the population that becomes ill?

Newton's Law of Cooling

The temperature, T, of a heated object at time t is given by $\boldsymbol{T = C + (T_0 - C)e^{kt}}$

Where: **C** is the constant temperature of the surrounding medium (often room temperature).

$\boldsymbol{T_0}$ is the initial temperature of the heated object.

$\boldsymbol{k}$ is a negative constant that is associated with cooling object.

Example: A cake removed from the oven has a temperature of 210°F. It is left to cool in a room that has a temperature of 70°F. After 30 minutes, the temperature of the cake is 140°F.

a. Use Newton's Law of Cooling to find a model for the temperature of the cake, T, after t minutes.

b. What is the temperature of the cake after 40 minutes?

c. When will the temperatures of the cake be 90°F?

Formulas you need to know from this chapter:

Compound Interest	$A = P\left(1 + \frac{r}{n}\right)^{nt}$
Continuous Compounding Exponential Growth/Decay	$A = P(e)^{rt}$
Logistic Growth	$A = \frac{c}{1 + a(e)^{-bt}}$
Newton's Law of Cooling	$T = C + (T_0 - C)e^{kt}$

Name: ______________________________ Date: ______________ Period: ______

Chapter 3: Exponential & Logarithmic Functions
Topic 5: Homework

The exponential models describe the population of the indicated country, A, in millions, t years after 2010. Use these models to solve Exercises 1–6.

India: $A = 1173.1e^{0.008t}$

Iraq: $A = 31.5e^{0.019t}$

Japan: $A = 127.3e^{-0.006t}$

Russia: $A = 141.9e^{-0.005t}$

1. What was the population of Japan in 2010?
2. What was the population of Iraq in 2010?
3. Which country has the greatest growth rate? By what percentage is the population of that country increasing each year?
4. Which countries have a decreasing population? By what percentage is the population of these countries decreasing each year?
5. When will India's population be 1377 million?
6. When will India's population be 1491 million?

In Exercises 21–26, complete the table. Round half-lives to one decimal place and values of k to six decimal places.

	Radioactive Substance	Half-Life	Decay Rate, k
21.	Tritium		5.5% per year $= -0.055$
22.	Krypton-85		6.3% per year $= -0.063$
23.	Radium-226	1620 years	
24.	Uranium-238	4560 years	
25.	Arsenic-74	17.5 days	
26.	Calcium-47	113 hours	

Use Newton's Law of Cooling, $T = C + (T_0 - C)e^{kt}$

A bottle of juice initially has a temperature of 70°F. It is left to cool in a refrigerator that has a temperature of 45°F. After 10 minutes, the temperature of the juice is 55°F.

a. Use Newton's Law of Cooling to find a model for the temperature of the juice, T, after t minutes.

b. What is the temperature of the juice after 15 minutes?

c. When will the temperature of the juice be 50°F?